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## UNIT-4

### M-TYPE TUBES

#### SCATTERING MATRIX:

"Scattering" is an idea taken from billiards, or pool. One takes a cue ball and fires it up the table at a collection of other balls. After the impact, the energy and momentum in the cue ball is divided between all the balls involved in the impact. The cue ball "scatters" the stationary target balls and in turn is deflected or "scattered" by them.

In a microwave circuit, the equivalent to the energy and momentum of the cue ball is the amplitude and phase of the incoming wave on a transmission line. (A rather loose analogy, this). This incoming wave is "scattered" by the circuit and its energy is partitioned between all the possible outgoing waves on all the other transmission lines connected to the circuit. The scattering parameters are fixed properties of the (linear) circuit which describe how the energy couples between each pair of ports or transmission lines connected to the circuit.

Formally, s-parameters can be defined for any collection of linear electronic components, whether or not the wave view of the power flow in the circuit is necessary. They are algebraically related to the impedance parameters (z-parameters), also to the admittance parameters (y-parameters) and to a notional characteristic impedance of the transmission lines.

#### COCEPT OF N PORT SCATTERING MATRIX REPRESENTATION:

*An n-port microwave network has n arms into which power can be fed and from which power can be taken. In general, power can get from any arm (as input) to any other arm (as output). There are thus n incoming waves and n outgoing waves.*

We also observe that power can be reflected by a port, so the input power to a single port can partition between all the ports of the network to form outgoing waves. Associated with each port is the notion of a "reference plane" at which the wave amplitude and phase is defined.

Usually the reference plane associated with a certain port is at the same place with respect to incoming and outgoing waves. The  $n$  incoming wave complex amplitudes are usually designated by the  $n$  complex quantities  $a_n$ , and the  $n$  outgoing wave complex quantities are designated by the  $n$  complex quantities  $b_n$ . The incoming wave quantities are assembled into an  $n$ -vector

$A$  and the outgoing wave quantities into an  $n$ -vector  $B$ . The outgoing waves are expressed in terms of the incoming waves by the matrix equation  $B = SA$  where  $S$  is an  $n$  by  $n$  square matrix of complex numbers called the "scattering matrix". It completely determines the behaviour of the network. In general, the elements of this matrix, which are termed "s-parameters", are all frequency-dependent.

For example, the matrix equations for a

$$\begin{aligned} \text{2-port are } b_1 &= s_{11} a_1 + s_{12} a_2 \\ b_2 &= s_{21} a_1 + s_{22} a_2 \end{aligned}$$

And the matrix equations for a 3-

$$\begin{aligned} \text{port are } b_1 &= s_{11} a_1 + s_{12} a_2 + s_{13} a_3 \\ b_2 &= s_{21} a_1 + s_{22} a_2 + s_{23} a_3 \\ b_3 &= s_{31} a_1 + s_{32} a_2 + s_{33} a_3 \end{aligned}$$

*The wave amplitudes  $a_n$  and  $b_n$  are obtained from the port current and voltages by the relations  $a = (V + Z_0 I) / (2 \sqrt{2 Z_0})$  and  $b = (V - Z_0 I) / (2 \sqrt{2 Z_0})$ . Here,  $a$  refers to an if  $V$  is  $V_n$  and  $I$  is  $I_n$  for the  $n$ th port. Note the  $\sqrt{2}$  reduces the peak value to an rms value, and the  $\sqrt{Z_0}$  makes the amplitude normalised with respect to power, so that the incoming power =  $aa^*$  and the outgoing power is  $bb^*$ .*

A one-port scattering parameter  $s$  is merely the reflection coefficient  $\gamma$ , and as we have seen we can relate  $\gamma$  to the load impedance  $z_L = Z_L / Z_0$  by the formula  $\gamma = (z_L - 1) / (z_L + 1)$ .

Similarly, given an  $n$  by  $n$  "Z-matrix" for an  $n$ -port network, we obtain the  $S$  matrix from the formula  $S = (Z - I)(Z + I)^{-1}$ , by post-multiplying the matrix  $(Z - I)$  by the inverse of the matrix  $(Z + I)$ . Here,  $I$  is the  $n$  by  $n$  unit matrix. The matrix of  $z$  parameters (which has  $n$  squared elements) is the inverse of the matrix of  $y$  parameters.

## PROPERTIES OF S MATRIX

### 1) Zero diagonal elements for perfect matched network

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For an ideal network with matched termination  $S_{ii}=0$ , since there is no reflection from any port.

Therefore under perfect matched condition the diagonal element of  $[s]$  are zero

2) Symmetry of  $[s]$  for a reciprocal network

The reciprocal device has a same transmission characteristics in either direction of a pair of ports and is characterized by a symmetric scattering matrix

$$S_{ij} = S_{ji} ; \quad i \neq j \text{ Which results } [S]^t = [S]$$

**For a reciprocal network with assumed normalized the impedance matrix equation is  $[b]$**

$$= ([Z] + [u])^{-1} ([Z] - [u]) [a] \text{ ----- (1)}$$

Where  $u$  is the unit matrix  $S$   
matrix equation of network is

$$[b] = [s] [a] \text{----- (2)}$$

Compare equ (1) & (2)

$$[s] = ([Z] + [u])^{-1} ([Z] - [u])$$

$$[R] = [Z] - [U]$$

$$[Q] = [Z] + [U]$$

For a reciprocal network  $Z$  matrix Symmetric

$$[R] [Q] = [Q] [R]$$

$$[Q]^{-1} [R] [Q] [Q]^{-1} = [Q]^{-1} [Q] [R] [Q]^{-1}$$

$$[ \quad -1$$

$$[ \quad -1 \text{ ----- (3)}$$

TRANSPOSE OS  $[s]$  IS NOW GIVEN AS

$$[S]^t = [Z-u]^t [$$

$$Z+U]^t^{-1}$$

**Then**

$$[Z-u]^t = [Z-U]$$

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$$[Z+u]t^{-1} = [$$

$$Z+U] [S] t = [z-u]$$

$$[z+u]^{-1}$$

$$[S]^{-1} = [R][Q]^{-1} \text{ ----- (4)}$$

**When compare 3 & 4 [S] t**

$$= [S]$$

**3) Unitary property of lossless network**

For any loss less network the sum of product of each term of any one row or any one column of s matrix multiplied by its complex conjugate is unity

N

$$\sum_{n=1}^N S_{ni} S_{ni}^* = 1$$

n=1

**For a lossless N port devices the total power leaving N ports must be equal to total input to the ports**

**4) Zero property**

It states that the sum of the product of any each term of any one row or any one column of a s matrix is multiplied by the complex conjugate of corresponding terms of any other row is zero

N

$$\sum_{n=1}^N S_{ni} S_{nj}^* = 0$$

**5) Phase shift propert**

If any of the terminal or reference plane are mover away from the junction by an electric distance  $\beta l_k$ , each of the coefficient  $S_{ij}$  involving K will be multiplied by the factor  $(e - j\beta l_k/k)$

$$S = \begin{pmatrix} 0 & e^{-j\phi_{12}} \\ e^{-j\phi_{21}} & 0 \end{pmatrix}$$

## **S MATRIX FORMULATION OF TWO PORT JUNCTION**

*In the case of a microwave network having two ports only, an input and an output, the s- matrix has four s-parameters, designated  $s_{11}$   $s_{12}$   $s_{21}$   $s_{22}$*

These four complex quantities actually contain eight separate numbers; the real and imaginary parts, or the modulus and the phase angle, of each of the four complex scattering parameters. Let us consider the physical meaning of these s-parameters. If the output port 2 is terminated, that is, the transmission line is connected to a matched load impedance giving rise to no reflections, and then there is no input wave on port 2.

The input wave on port 1 ( $a_1$ ) gives rise to a reflected wave at port 1 ( $s_{11}a_1$ ) and a transmitted wave at port 2 which is absorbed in the termination on 2.

The transmitted wave size is ( $s_{21}a_1$ ). If the network has no loss and no gain, the output power must equal the input power and so in this case  $|s_{11}|^2 + |s_{21}|^2$  must equal unity. We see therefore that the sizes of  $S_{11}$  and  $S_{21}$  determine how the input power splits between the possible output paths.

### **MICROWAVE JUNCTIONS:**

E  
PLANE  
TEE

H  
PLANE  
TEE

MAGIC TEE OR HYPRID TEE  
**TEE JUNCTIONS:**

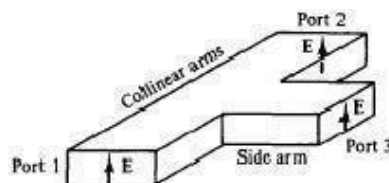
**Tee junctions.** In microwave circuits a waveguide or coaxial-line junction with three independent ports is commonly referred to as a *tee junction*.

From the  $S$  parameter theory of a microwave junction it is evident that a tee junction should be characterized by a matrix of third order containing nine elements, six of which should be independent.

The characteristics of a three-port junction can be explained by three theorems of the tee junction. These theorems are derived from the equivalent- circuit representation of the tee junction. Their statements follow

- 1. A short circuit may always be placed in one of the arms of a three-port junction in such a way that no power can be transferred through the other two arms.**
- 2. If the junction is symmetric about one of its arms, a short circuit can always be placed in that arm so that no reflections occur in power transmission between the other two arms. (That is, the arms present matched impedances.)**
- 3. It is impossible for a general three-port junction of arbitrary symmetry to present matched impedances at all three arms.**

**H-plane tee (shunt tee).** An  $H$ -plane tee is a waveguide tee in which the axis of its side arm is "shunting" the  $E$  field or parallel to the  $H$  field of the main guide as shown in Fig.



It can be seen that if two input waves are fed into port 1 and port 2 of the collinear arm, the output wave at port 3 will be in phase and additive. On the other hand, if the input is fed into port 3, the wave will split equally into port 1 and port 2 in phase and in the same magnitude.



***E* -plane tee (series tee).** An *E* -plane tee is a waveguide tee in which the axis of its side arm is parallel to the *E* field of the main guide

If the collinear arms are symmetric about the side arm, there are two different transmission characteristics

It can be seen from Fig. 4-4-4 that if the *E*-plane tee is perfectly matched with the aid of screw tuners or inductive or capacitive windows at the junction, the diagonal components of the scattering matrix,  $S_{11}$ ,  $S_{22}$ , and  $S_{33}$ , are zero because there will be no reflection.

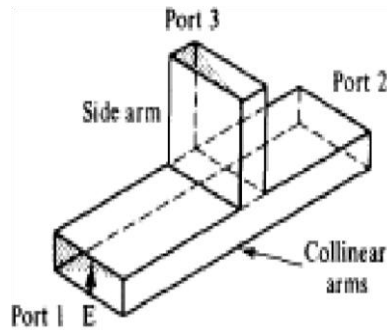


Figure 4-4-4 *E*-plane tee

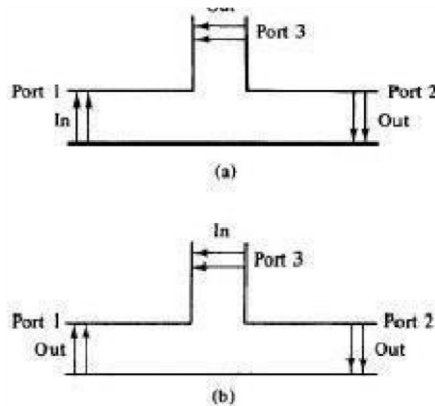


Figure 4-4-5 Two-way transmission of *E*-plane tee. (a) Input through main arm. (b) Input from side arm.

is always negative. The negative sign merely means that  $S_{13}$  and  $S_{23}$  have opposite signs. For a matched junction, the *S* matrix is given by

$$S = \begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{21} & 0 & S_{23} \\ S_{31} & S_{32} & 0 \end{bmatrix} \quad (4-4-13)$$

$$S_{12} = S_{21} \quad S_{13} = S_{31} \quad S_{23} = S_{32} \quad (4-4-14)$$

From the symmetry property of *S* matrix, the symmetric terms in Eq. (4-4-13) are equal and they are

From the zero property of S matrix, the sum of the products of each term of any column (or row) multiplied by the complex conjugate of the corresponding terms of any other column (or row) is zero and it is

Hence

$$S_{11} S_{12}^* + S_{21} S_{22}^* + S_{31} S_{32}^* = 0 \quad (4-4-15)$$

$$S_{12} S_{23}^* = 0 \quad (4-4-16)$$

This means that either  $S_{13}$  or  $S_{t3}$ , or both, should be zero. However, from the unity property of S matrix, the sum of the products of each term of any one row (or column) multiplied by its complex conjugate is unity; that is,

$$S_{21} S_{21}^* + S_{31} S_{31}^* = 1 \quad (4-4-17)$$

$$S_{12} S_{12}^* + S_{32} S_{32}^* = 1 \quad (4-4-18)$$

$$S_{13} S_{13}^* + S_{23} S_{23}^* = 1 \quad (4-4-19)$$

Substitution of Eq. ( 4-4-I4) in ( 4-4-I7) results in

$$|S_{12}|^2 = 1 - |S_{13}|^2 = 1 - |S_{23}|^2 \quad (4-4-20)$$

zero and thus Eq. ( 4-4-19) is false. In a similar fashion, if  $S_{23} = 0$ , then  $S_{13}$  becomes zero and therefore Eq. (4-4-20) is not true.

This inconsistency proves the statement that the tee junction cannot be matched to the three arms. In other words, the diagonal elements of the S matrix of a tee junction are not all zeros.

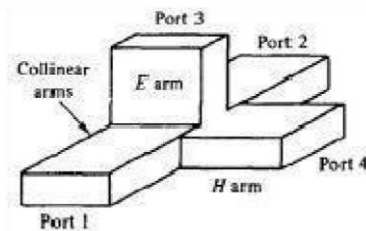
In general, when an E-plane tee is constructed of an empty waveguide, it is poorly matched at the tee junction. Hence  $S_{ij} \neq 0$  if  $i = j$ .

However, since the collinear arm is usually symmetric about the side arm,  $|S_{13}| = |S_{23}|$  and  $S_{11} = S_{22}$ . Then the S matrix can be simplified to

$$S = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{11} & -S_{13} \\ S_{13} & -S_{13} & S_{13} \end{bmatrix} \quad (4-4-21)$$

**MAGIC TEE:**

A magic tee is a combination of the E-plane tee and H-plane tee (refer to Fig. 4-4-7). The magic tee has several characteristics:



1. If two waves of equal magnitude and the same phase are fed into port 1 and port 2, the output will be zero at port 3 and additive at port 4.
2. If a wave is fed into port 4 (the Harm), it will be divided equally between port 1 and port 2 of

the collinear arms and will not appear at port 3 (the E arm).

3. If a wave is fed into port 3 (the E arm), it will produce an output of equal magnitude and opposite phase at port 1 and port 2. The output at port 4 is zero. That is,  $S_{43} = S_{34} = 0$ .
4. If a wave is fed into one of the collinear arms at port 1 or port 2, it will not appear in the other collinear arm at port 2 or port 1 because the *E* arm causes a phase delay while the *H* arm causes a phase advance. That is,  $S_{z1} = S_{1z} = 0$ .

Therefore the S matrix of a magic tee can be expressed as

The magic tee is commonly used for mixing, duplexing, and impedance measurements. Suppose, for example, there are two identical radar transmitters in equipment stock.

A particular application requires twice more input power to an antenna than either transmitter can deliver. A magic tee may be used to couple the two transmitters to the antenna in such a way that the transmitters do not load each other.

The two transmitters should be connected to ports 3 and 4, respectively, as shown in Fig. 4-4-8. Transmitter 1, connected to port 3, causes a wave to emanate from port 1 and another to emanate from port 2; these waves are equal in magnitude but opposite in phase.

Similarly, transmitter 2, connected to port 4, gives rise to a wave at port 1 and another at port 2, both equal in magnitude and in phase.

At port 1 the two opposite waves cancel each other. At port 2 the two in-phase waves add together; so double output power at port 2.

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & S_{24} \\ S_{31} & S_{32} & S_{33} & S_{34} \\ S_{41} & S_{42} & S_{43} & S_{44} \end{bmatrix}$$

But  $S_{21} = 0, S_{12} = 0, S_{43} = 0, S_{34} = 0$

$S_{11} = 0, S_{22} = 0, S_{33} = 0, S_{44} = 0$

and  $S_{14} = S_{24}, S_{13} = -S_{23}$

For port-3 and port-4 matched

∴ S-matrix becomes

$$[S] = \begin{bmatrix} 0 & 0 & S_{13} & S_{14} \\ 0 & 0 & -S_{13} & S_{14} \\ S_{31} & S_{32} & 0 & 0 \\ S_{41} & S_{42} & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & S_{13} & S_{13} \\ 0 & 0 & -S_{13} & S_{13} \\ S_{13} & -S_{13} & 0 & 0 \\ S_{13} & S_{13} & 0 & 0 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & -1 & 1 \\ 1 & -1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix}$$

## RATE RACE –CORNERS

Applications of rat-race couplers are numerous, and include mixers and phase shifters. The rat-race gets its name from its circular shape, shown below. The circumference is 1.5 wavelengths. For an equal-split rat-race coupler, the impedance of the entire ring is fixed at  $1.41 \times Z_0$ , or 70.7 ohms for a 50 ohm system. For an input signal  $V_{in}$ , the outputs at ports 2 and 4 (thanks, Tom!) are equal in magnitude, but 180 degrees out of phase.

The coupling of the two arms is shown in the figure below, for an ideal rat-race coupler centered at 10 GHz (10,000 MHz). An equal power split of 3 dB occurs at only the center frequency. The 1-dB bandwidth of the coupled port (S41) is shown by the markers to be 3760 MHz, or 37.6 percent.

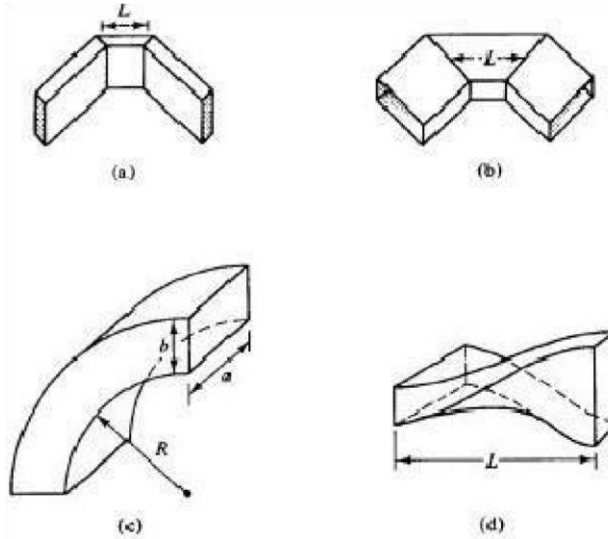
## BENTS & TWISTS:

The waveguide corner, bend, and twist are shown in Fig. 4-4-10. These waveguide components are normally used to change the direction of the guide through an arbitrary angle. In order to minimize reflections from the discontinuities, it is desirable to have the mean length  $L$  between continuities equal to an odd number of quarter-wavelengths. That is,

where  $n = 0, 1, 2, 3, \dots$ , and  $\lambda$  is the wavelength in the waveguide. If the mean length  $L$  is an odd number of quarter wavelengths, the reflected waves from both ends of the waveguide section are completely canceled. For the waveguide bend, the minimum radius of curvature for a small reflection is given by Southworth [2] as

$$R = 1.5b \text{ for an } E \text{ bend}$$

$$R = 1.5a \text{ for an } H \text{ bend}$$



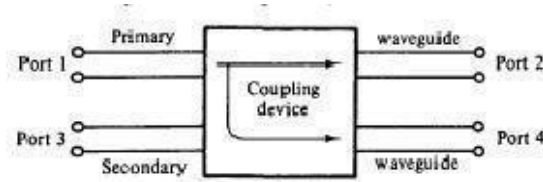
**DIRECTIONAL COUPLERS:**

**A directional coupler is a four-port waveguide junction as shown in Fig. 4-5-1. It consists of a primary waveguide 1-2 and a secondary waveguide 3-4.**

When all ports are terminated in their characteristic impedances, there is free transmission of power, without reflection, between port 1 and port 2, and there is no transmission of power between port 1 and port 3 or between port 2 and port 4 because no coupling exists between these two pairs of ports.

The degree of coupling between port 1 and port 4 and between port 2 and port 3 depends on the structure of the coupler. The characteristics of a directional coupler can be expressed in terms of its coupling factor and its directivity.

Assuming that the wave is propagating from port 1 to port 2 in the primary line, the coupling factor and the directivity are defined,



respectively, by

$$\text{Coupling factor (dB)} = 10 \log_{10} \frac{P_1}{P_3}$$

$$\text{Directivity (dB)} = 10 \log_{10} \frac{P_4}{P_3}$$

where  $P_1$  = power input to port 1

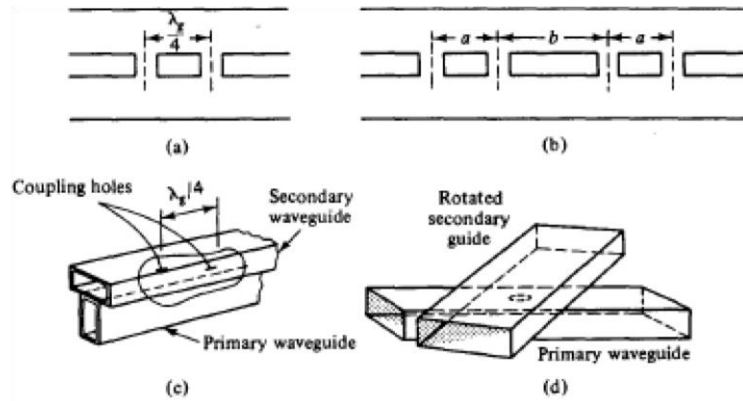
$P_3$  = power output from port 3

$P_4$  = power output from port 4

It should be noted that port 2, port 3, and port 4 are terminated in their characteristic impedances. *The coupling factor is a measure of the ratio of power levels in the primary and secondary lines.* Hence if the coupling factor is known, a fraction of power measured at port 4 may be used to determine the power input at port

4. This significance is desirable for microwave power measurements because no disturbance, which may be caused by the power measurements, occurs in the primary line.
5. The directivity is a measure of how well the forward traveling wave in the primary waveguide couples only to a specific port of the secondary waveguide. An ideal directional coupler should have infinite directivity. In other words, the power at port 3 must be zero because port 2 and port 4 are perfectly matched.
6. Actually, well-designed directional couplers have a directivity of only 30 to 35 dB. Several types of directional couplers exist, such as a two-hole directional coupler, four-hole directional coupler, reverse-coupling directional coupler (Schwinger coupler), and Bethe-hole directional coupler (refer to Fig. 4-5-2). Only the very commonly used two-hole directional coupler is described here.





**Figure 4-5-2** Different directional couplers. (a) Two-hole directional coupler. (b) Four-hole directional coupler. (c) Schwinger coupler. (d) Bethe-hole directional coupler.

$$S_{11} = S_{22} = S_{33} = S_{44} = 0$$

As noted, there is no coupling between port 1 and port 3 and between port 2 and port 4. Thus

$$S_{13} = S_{31} = S_{24} = S_{42} = 0$$

Consequently, the S matrix of a directional coupler becomes

$$\mathbf{S} = \begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{21} & 0 & S_{23} & 0 \\ 0 & S_{32} & 0 & S_{34} \\ S_{41} & 0 & S_{43} & 0 \end{bmatrix}$$

Equation (4-5-6) can be further reduced by means of the zero property of the S matrix, so we have

$$S_{12}S_{14}^* + S_{32}S_{34}^* = 0$$

$$S_{21}S_{23}^* + S_{41}S_{43}^* = 0$$

Also from the unity property of the S matrix, we can write

$$S_{12}S_{12}^* + S_{14}S_{14}^* = 1$$

Equations (4-5-7) and (4-5-8) can also be written

$$|S_{12}| |S_{14}| = |S_{32}| |S_{34}|$$

$$|S_{21}| |S_{23}| = |S_{41}| |S_{43}|$$

Since  $S_{12} = S_{21}$ ,  $S_{14} = S_{41}$ ,  $S_{23} = S_{32}$ , and  $S_{34} = S_{43}$ , then

$$|S_{12}| = |S_{34}|$$

$$|S_{14}| = |S_{23}|$$

Let

$$S_{12} = S_{34} = p$$

where  $p$  is positive and real. Then from Eq. (4-5-8)

$$p(S_{23}^* + S_{43}) = 0$$

Let

$$S_{23} = S_{43} = jq$$

where  $q$  is positive and real. Then from Eq. (4-5-9)

$$p^2 + q^2 = 1$$

The S matrix of a directional coupler is reduced to

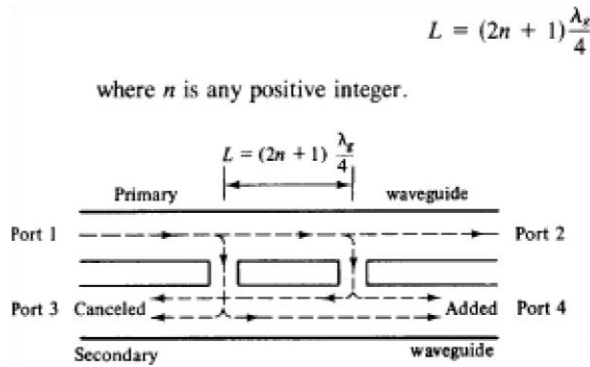
$$\mathbf{S} = \begin{bmatrix} 0 & p & 0 & jq \\ p & 0 & jq & 0 \\ 0 & jq & 0 & p \\ jq & 0 & p & 0 \end{bmatrix}$$

## TWO HOLE DIRECTIONAL COUPLERS:

### Two-Hole Directional Couplers

A two-hole directional coupler with traveling waves propagating in it is illustrated in Fig. 4-5-3.

The spacing between the centers of two holes must be



A fraction of the wave energy entered into port 1 passes through the holes and is radiated into the secondary guide as the holes act as slot antennas.

The forward waves in the secondary guide are in the same phase, regardless of the hole space, and are added at port 4.

The backward waves in the secondary guide (waves are progressing from right to left) are out of phase by  $(2L/\lambda_g)\pi$  rad and are canceled at port 3.

In a directional coupler all four ports are completely matched. Thus the diagonal elements of the S matrix are zeros

**FERRITES:**

An *isolator* is a nonreciprocal transmission device that is used to isolate one component from reflections of other components in the transmission line. An ideal isolator completely absorbs the power for propagation in one direction and provides lossless transmission in the opposite direction.

Thus the isolator is usually called *uniline*. Isolators are generally used to improve the frequency stability of microwave generators, such as klystrons and magnetrons, in which the reflection from the load affects the generating frequency.

In such cases, the isolator placed between the generator and load prevents the reflected

power from the unmatched load from returning to the generator. As a result, the isolator maintains the frequency stability of the generator. Isolators can be constructed in many ways.

They can be made by terminating ports 3 and 4 of a four-port circulator with matched loads. On the other hand, isolators can be made by inserting a ferrite rod along the axis of a rectangular waveguide as shown in Fig. 4-6-5. The isolator here is a Faraday-rotation isolator. Its operating principle can be explained as follows [5]. The input resistive card is in the  $y$ - $z$  plane, and the output resistive card is displaced  $45^\circ$  with respect to the input card.

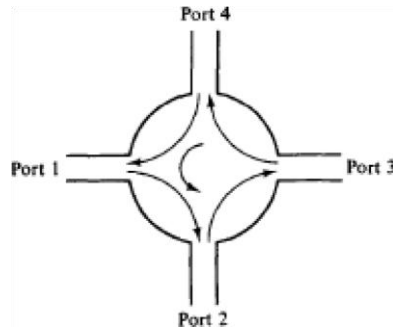
The de magnetic field, which is applied longitudinally to the ferrite rod, rotates the wave plane of polarization by  $45^\circ$ . The degrees of rotation depend on the length and diameter of the rod and on the applied de magnetic field. An input TE<sub>10</sub> dominant mode is incident to the left end of the isolator. Since the TE<sub>10</sub> mode wave is perpendicular to the input resistive card, the wave passes through the ferrite rod without attenuation.

The wave in the ferrite rod section is rotated clockwise by  $45^\circ$  and is normal to the output resistive card. As a result of rotation, the wave arrives at the output

#### *TERMINATION:*

A *microwave circulator* is a multiport waveguide junction in which the wave can flow only from the  $n$ th port to the  $(n + 1)$ th port in one direction

Although there is no restriction on the number of ports, the four-port microwave circulator is the most common. One type of four-port microwave circulator is a combination of two 3-dB side-hole directional couplers and a rectangular waveguide with two nonreciprocal phase shifters as shown in Fig



The operating principle of a typical microwave circulator can be analyzed with the aid of Fig. Each of the two 3-dB couplers in the circulator introduces a phase shift of  $90^\circ$ , and each of the two phase shifters produces a certain amount of phase change in a certain direction as indicated.

When a wave is incident to port 1, the wave is split into two components by coupler 1. The wave in the primary guide arrives at port 2 with a relative phase change of  $180^\circ$ . The second wave propagates through the two couplers and the secondary guide and arrives at port 2 with a relative phase shift of  $180^\circ$ . Since the two waves reaching port 2 are in phase, the power transmission is obtained from port 1 to port 2.

However, the wave propagates through the primary guide, phase shifter, and coupler 2 and arrives at port 4 with a phase change of  $270^\circ$ . The wave travels through coupler 1 and the secondary guide, and it arrives at port 4 with a phase shift of  $90^\circ$ . Since the two waves reaching port 4 are out of phase by  $180^\circ$ , the power transmission from port 1 to port 4 is zero. In general, the differential propagation constants in the two directions of propagation in a waveguide containing ferrite phase shifters should be

$$\omega_1 - \omega_3 = (2m + 1)\pi \quad \text{rad/s}$$

$$\omega_2 - \omega_4 = 2n\pi \quad \text{rad/s}$$

*GYRATOR:*

A **gyrator** is a passive, linear, lossless, two-port electrical network element proposed in 1948 by Bernard H. Tellegen as a hypothetical fifth linear element after the resistor, capacitor, inductor and ideal transformer. Unlike the four conventional elements, the gyrator is non-

reciprocal. Gytrators permit network realizations of two-(or-more)-port devices which cannot be realized with just the conventional four elements.

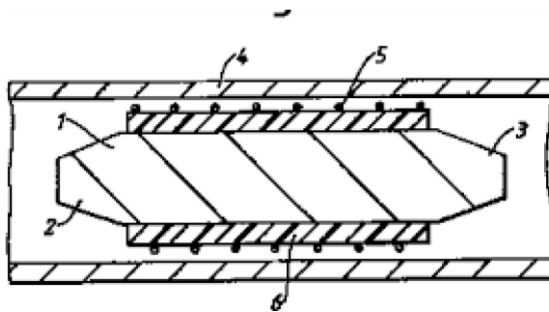
In particular, gytrators make possible network realizations of isolators and circulators.

Gytrators do not however change the range of one-port devices that can be realized. Although the gytrator was conceived as a fifth linear element, its adoption makes both the ideal transformer and either the capacitor or inductor

redundant. Thus the number of necessary linear elements is in fact reduced to three. Circuits that function as gytrators can be built with transistors and op amps using feedback.

W. C. C. ellegen invented a circuit symbol for the gytrator and suggested a number of ways in which a practical gytrator might be built.

An important property of a gytrator is that it inverts the current-voltage characteristic of an electrical component or network. In the case of linear elements, the impedance is also inverted. In other words, a gytrator can make a capacitive circuit behave inductively, a series LC circuit behave like a parallel LC circuit, and so on. It is primarily used in active filter design and miniaturization.



### **ISOLATOR CIRCULATOR:**

An *isolator* is a nonreciprocal transmission device that is used to isolate one component from reflections of other components in the transmission line. An ideal isolator completely absorbs the power for propagation in one direction and provides lossless transmission in the opposite direction. Thus the isolator is usually called *uniline*.

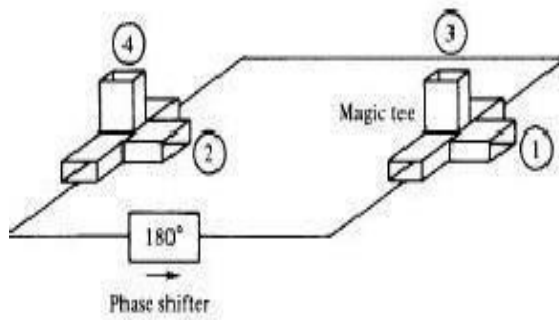
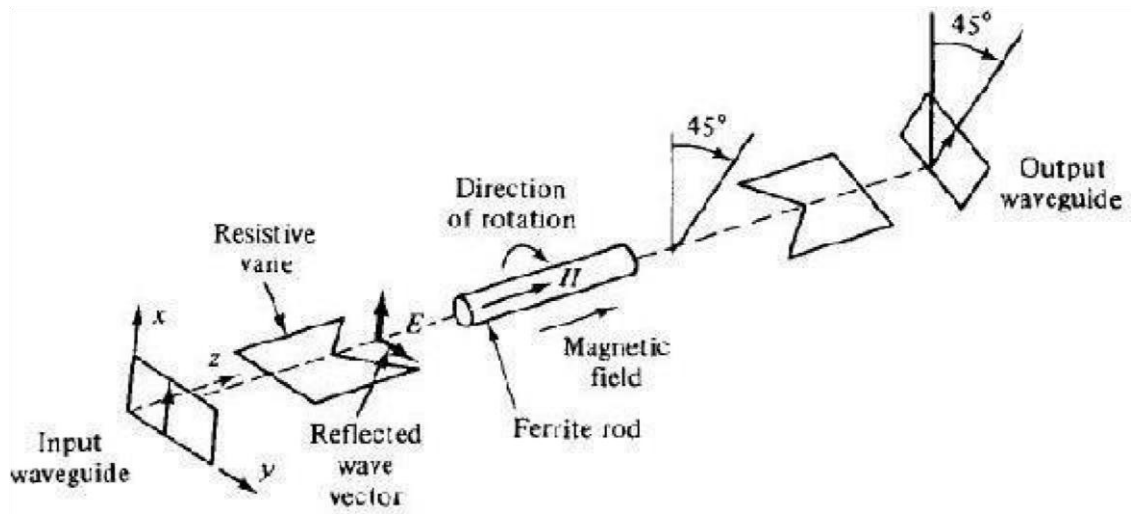
Isolators are generally used to improve the frequency stability of microwave generators, such as klystrons and magnetrons, in which the reflection from the load affects the generating frequency. In such cases, the isolator placed between the generator and load prevents the reflected power from the unmatched load from returning to the generator.

As a result, the isolator maintains the frequency stability of the generator. Isolators can be constructed in many ways. They can be made by terminating ports 3 and 4 of a four-port circulator with matched loads. On the other hand, isolators can be made by inserting a ferrite rod along the axis of a rectangular waveguide as shown in Fig. 4-6-5.

The isolator here is a Faraday-rotation isolator. Its operating principle can be explained as follows [5]. The input resistive card is in the  $y$ - $z$  plane, and the output resistive card is displaced  $45^\circ$  with respect to the input card. The de magnetic field, which is applied longitudinally to the ferrite rod, rotates the wave plane of polarization by  $45^\circ$ . The degrees of rotation depend on the length and diameter of the rod and on the applied de magnetic field. An input TE<sub>10</sub> dominant mode is incident to the left end of the isolator. Since the TE<sub>10</sub> mode wave is perpendicular to the input resistive card, the wave passes through the ferrite rod without attenuation.

The wave in the ferrite rod section is rotated clockwise by  $45^\circ$  and is normal to the output resistive card. As a result of rotation, the wave arrives at the output end without attenuation at all. On the contrary, a reflected wave from the output end is similarly rotated clockwise  $45^\circ$  by the ferrite rod.

However, since the reflected wave is parallel to the input resistive card, the wave is thereby absorbed by the input card. The typical performance of these isolators is about 1-dB insertion loss in forward transmission and about 20- to 30-dB isolation in reverse attenuation





$$\mathbf{S} = \begin{bmatrix} 0 & S_{12} & S_{13} & S_{14} \\ S_{21} & 0 & S_{23} & S_{24} \\ S_{31} & S_{32} & 0 & S_{34} \\ S_{41} & S_{42} & S_{43} & 0 \end{bmatrix}$$

$$\mathbf{S} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

72 PHASE CHANGER:

### S MATRIX FOR MICROWAVE COMPONENTS:

**H palne tee:**

$$[S] = \begin{bmatrix} 1/2 & -1/2 & 1/\sqrt{2} \\ -1/2 & 1/2 & 1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \end{bmatrix}$$

Circulator:

$$\mathbf{S} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Directional coupler:

$$\mathbf{S} = \begin{bmatrix} 0 & p & 0 & jq \\ p & 0 & jq & 0 \\ 0 & jq & 0 & p \\ jq & 0 & p & 0 \end{bmatrix}$$

*E palne Tee*

$$\mathbf{S} = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{11} & -S_{13} \\ S_{13} & -S_{13} & S_{33} \end{bmatrix} \quad (4-4-21)$$